PREFACE

Special section on
Mathematical Modeling in Medical Problems

For many years, mathematical modeling has been successfully used in many fields of science, such as physics, economics, biology, and chemistry. Recently, mathematicians and computer scientists have also addressed challenging questions in medicine. This special paper collection is focused on current advances in the mathematical modeling of medical problems. Mathematical models help to explain a disease's course and deepen our understanding of many interacting processes involved in its development. Consequently, the analysis of such models allows us to answer important questions posed by clinicians or doctors. Since obtaining analytical solutions to such problems is often a time-consuming or even impossible task, computational methods may be necessary to provide a solution. This includes the use of numerical methods of solving differential equations or computational simulations of agent-based models, which depend on the initial data and other model parameters.

This special section aims to showcase the wide spectrum of applications of mathematical models in biomedical problems. Six papers related to various aspects of such applications are presented.

The first work, entitled *Applications of the fractional Sturm–Liouville difference problem to the fractional diffusion difference equation*, is the most theoretical one. The authors focus on the description and analysis of the fractional diffusion difference equations which can be applied in the analysis of anomalous diffusion. Such a type of diffusion has been recognized in the motion of substances occurring in cell biology. In contrast to normal diffusion, a non-linear relation between the mean square displacement and time is assumed in anomalous diffusion. Both homogeneous and non-homogeneous equations with the operators defined in the sense of Grünwald–Letnikov are considered in this paper. It is proved that solutions exist and are given by the finite series.

In contrast, the next paper, *Assessment measures of an ensemble classifier based on the distributivity equation to predict the presence of severe coronary artery disease*, is based on clinical data related to coronary artery disease, 24-hour Holter ECG monitoring, and coronary angiography. Using these data the authors evaluate the usefulness of the hybrid classifier to predict the presence of the disease. Their hybrid algorithm is created from aggregation. The results of comparing this approach with the distributive equation of selected classification algorithms are presented.

The third contribution, *On the analysis of a mathematical model of CAR-T cell therapy for glioblastoma: Insights from a mathematical model*, is related to a novel therapy of solid tumors, namely, CAR-T cell therapy analyzed in the context of brain tumors. The authors consider a system of non-linear differential equations (ODEs) describing interactions between tumor cells and engineered chimeric antigen receptor T (called CAR-T) cells injected into a patient's body. Two types of treatment are studied: constant (as a simplest mathematical approximation reflected by ODEs) and impulsive (reflected by impulsive differential equations) using parameters related to glioblastoma.

The fourth paper, entitled *Spike patterns and chaos in a map-based neuron model*, is devoted to the study of the piecewise linear version of the one-dimensional Courbage–Nekorkin–Vdovin model of a single neuron. The authors discuss both periodic and chaotic behavior of the system, with particular emphasis on the analysis of spike patterns.

The aim of the next article, *A method of lower and upper solutions for control problems and application to a model of bone marrow transplantation*, is to put forward an algorithm for the approximation of the solutions of general control problems. The authors prove the convergence of the algorithm and provide application of the theory to real-life problems encountered in biology.

The last paper, *Semi-supervised vs. supervised learning for mental health monitoring: A case study on bipolar disorder*, compares supervised and semi-supervised machine learning algorithms in the medical domain. The main goal of the manuscript is to verify if the inner structure of data improves the classification performance. A study is provided on real-life acoustic data referring to patients affected by bipolar disorder, where a small portion of data is labeled in the large amount of available data.

The editors hope that this special section and the presented applications of recently developed mathematical and numerical methods will be of interest to a wide range of scientists from various fields. We would like to thank all the authors who submitted their research papers to our special section for their hard work on making this publication full of interesting results, both from a theoretical and an application point of view. We also want to thank the reviewers—we believe that their comments and suggestions improved the manuscripts under consideration. We also wish to acknowledge

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